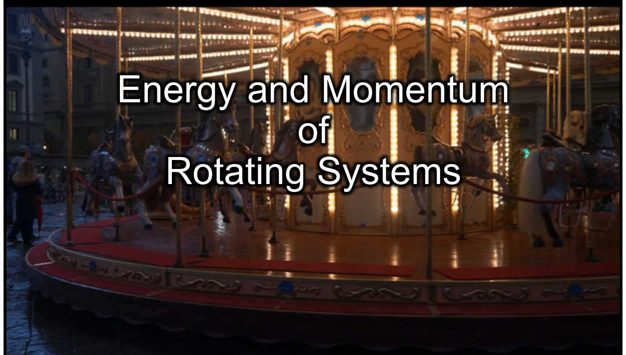


Energy and Momentum of Rotating Systems

BreakDownPictures (Pixabay)

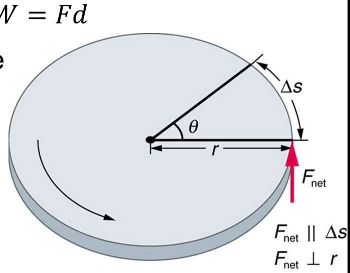
Rotational Kinetic Energy

- Work must be done to rotate objects.

$$W = Fd$$

- When a net force is applied perpendicular to the lever arm, the object rotates through an arc length Δs.

$$W = F\Delta s$$



$$\tau = rF \qquad W = \tau \frac{\Delta s}{r} \qquad \theta = \frac{\Delta s}{r}$$

$$W = \tau\theta \qquad \tau = I\alpha$$

$$W = I\alpha\theta \qquad \omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\alpha\theta = \frac{(\omega^2 - \omega_0^2)}{2}$$

$$W = \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0^2$$

- This is the work-energy theorem for rotational motion.

- Through an analogy with translational motion, we define the term $\frac{1}{2}I\omega^2$ to be rotational kinetic energy for an object with a moment of inertia I and an angular velocity ω.

$$\boxed{K = \frac{1}{2}I\omega^2}$$

Rolling and Slipping

- If an object makes a perfectly frictionless contact with a surface it is impossible for the object to roll – it simply slides.
- When there is friction the object can roll.
- Since the point of contact between the rolling body and the surface on which it rolls is instantaneously stationary, the coefficient of static friction should be used in calculations involving rolling.
- The point of contact must be stationary because it does not slide.

Example

A ball rolls down a ramp as shown without slipping. The moment of inertia of the ball is $\frac{2}{3}mr^2$. Calculate the linear velocity at the bottom of the ramp.

Energy is conserved.

$$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 \qquad v = \omega r$$

$$mgh = \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2$$

$$gh = \frac{v^2}{3} + \frac{v^2}{2} = \frac{5v^2}{6}$$

$$v = \sqrt{\frac{6gh}{5}}$$

$$v = \sqrt{\frac{6(9.8)(0.25)}{5}} = 1.7\ \mathrm{m/s}$$

Angular Momentum

- Angular momentum is defined as the product of the moment of inertia and the angular velocity.

$$\boxed{L = I\omega}$$

Units: kg m^2 rad/s

- A torque is required to make an object rotate.
- If the torque you exert is greater than opposing torques, then the rotation accelerates, and angular momentum increases.
- The greater the net torque, the more rapid the increase in L.
- The relationship between torque and angular momentum is

$$\boxed{\Delta L = \tau \Delta t}$$

(This is the rotational form of Newton's second law.)

Conservation of Angular Momentum

- The total angular momentum of a system remains constant providing no external torque acts on it.

Example

Suppose a star of radius R_1 has a period of 20 days. The star collapses to a radius R_2 which is smaller by a factor of 10000 without losing mass. What is the new period of the star?

Moment of inertia of a sphere = $\frac{2}{5}mr^2$

Angular momentum is conserved.

$$L_1 = L_2$$

$$I_1\omega_1 = I_2\omega_2$$

$$I = \frac{2}{5}mr^2 \qquad \omega = \frac{2\pi}{T}$$

$$\frac{2}{5}mR_1^2\left(\frac{2\pi}{T_1}\right) = \frac{2}{5}mR_2^2\left(\frac{2\pi}{T_2}\right)$$

$$R_1 = 10000R_2$$

$$T_2 = T_1\left(\frac{R_2}{10000R_2}\right)^2$$

$$T_2 = 20\left(\frac{1}{1\times10^4}\right)^2 = 2\times10^{-7}\ \text{days} = 0.017\ \text{s}$$